Children's Difficulties with Mathematical Word Problems.

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This article reports a study of the difficulties that primary school children experience whilst tackling school mathematical word-problems. A case study of four Year 5 children was conducted; this involved interviews which probed the children's views of their own difficulties and discussions with the children as they tackled word problems. The data were qualitatively analysed using a thematic analysis approach based on categories of difficulty identified from existing literature. Examples of transcripts and responses which show the children experiencing difficulties are included, as well as the children's opinions on their difficulties. My interpretation of these findings, including proposed subcategories of difficulty, is also given. The report concludes with suggestions of methods – subject to further research – that teachers may use to help children overcome their difficulties with school mathematical word problems.

Background

Children's poor performance with mathematical word problems is a trend that I became aware of very early on in my teaching career and one that an interest has been taken in by many who are involved in Mathematics education.
By looking at the existing literature on children's difficulties with mathematical word problems, I was able to gain a more detailed insight into the causes of children's difficulties. Using the evidence from existing research, I formulated five categories of difficulties that children may experience whilst tackling mathematical word problems. These categories are presented below.

Reading and Understanding the Language Used Within a Word Problem

Difficulties in this category involve children not being able to decode the words used in a word problem, not comprehending a sentence, not understanding specific vocabulary and not having confidence or the ability to concentrate when reading. (Ballew and Cunningham 1982: Shuard and Rothery 1984: Cummins et al 1988: Bernardo 1999).

Recognising and Imagining the Context in Which a Word Problem is Set

These difficulties arise when children cannot imagine the context in which a word problem is set or their approach is altered by the context in which the word problem is given. (Caldwell and Goldin 1979: Nunes 1993).

Forming a Number Sentence to Represent the Mathematics Involved in the Word Problem

Children appear to find it harder to form a number sentence for some word problems structures than others. These difficulties can result in children not being able to select a calculation to perform or selecting an incorrect calculation. (Carey 1991: English 1998).

Carrying Out the Mathematical Calculation

Difficulties can occur here with children's selection of, and aptitude with calculation strategies (for example formal algorithms, pencil and paper methods and calculators). The context in which a word problem is given and the size of numbers involved can impact on children's choice of a calculation strategy. (Verschaffel, De Corte and Vierstraete 1999: Nunes 1993: Anghileri 2001).

Interpreting the Answer in the Context of the Question

Children have been shown to not consider real-life factors and constraints when giving an answer to word problems which can result in giving an answer that is impossible in the context and therefore incorrect. (Verschaffel, De Corte and Lasure 1994; Wyndham and Säljö 1997; Cooper and Dunne 2000).

Method

Aims

The aims of the study were to establish whether difficulties within the identified categories occur in English Primary Schools and, if they do, to find examples of children experiencing difficulties within the categories. I hoped that examples of children experiencing the range of difficulties may provide a resource for increasing teachers' awareness of the difficulties.

Data Collection

Four children were selected to take part in the study. These children were from the Year 5 class that I taught; hence, I knew them well. They were selected on the criteria that they were willing and able to discuss the mathematics that they were doing and were working at a range of attainment levels in Mathematics lessons.

The first element of the data collection involved interviews on the children's views of difficulties they had experienced with mathematical word problems.

The second element involved the children working individually through sets of equivalent word problems and discussing their processes and difficulties with me. There were five sets of equivalent word problems that each child attempted. Each set was given in a different condition, with a different form of help given in each. Each form of help corresponded to one of the previously identified category of difficulty. For example I read the word problem to the child, explained any vocabulary and simplified sentences in condition one to correspond to the first category. I offered forms of help in the belief that if I gave a specific type of help and children then solved a problem, I could identify where the original difficulty lay and be aware of which kinds of help allow children to overcome certain types of difficulty.

Data analysis

The first stage of data analysis involved analysing and coding interview transcripts and recordings. Excerpts were coded under a category of difficulty if they showed opinions on that difficulty, a child experiencing that difficulty, or a child competently completing a process, therefore not having that difficulty. Any un-coded data were then checked for a need for new categories or reported as 'Other Findings'.

The second stage of data analysis involved analysing and coding all incorrect or no responses to word problems. Transcripts and children's jottings or workings were used to code as to which difficulty prevented a correct answer being given.

Using the coded data, I was able to create subcategories within some of the five main previously identified categories of difficulty.

Finally, I selected illustrative examples of children experiencing difficulties or giving opinions on difficulties from each category and subcategory. Examples were picked using the criteria of being typical and not extreme.

The Results and Discussion

The finalised categories and subcategories of difficulties formed are:

- Reading and Comprehension
  - Decoding the Words in a Word-Problem
  - Understanding the Meaning of the Words and Sentences
- Reading All of the Information
- Distracting Information
- Imagining the Context
- Writing a Number Sentence
- Carrying Out the Calculation
  - Lack of Accurate Methods for Calculating
  - Making a Mistake When Calculating
- Interpreting the Answer in the Context of the Question
  - Giving an Answer that is Possible or Likely
  - Transferring an Answer into the Required Format

I have selected examples of children experiencing the above difficulties, or opinions on difficulties from the categories 'Reading and Comprehension' and 'Interpreting the Answer in the Context of the Question' to present below.

Reading All of the Information

The following example shows Liam giving an incorrect response to a word problem because he has not read or comprehended all of the text in the question.

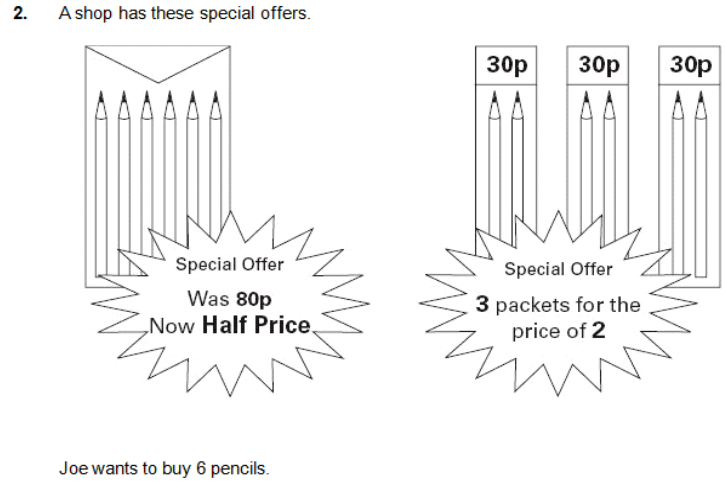

Figure 1, a word problem given to Liam and the transcript of the conversation that followed.

Liam: This is a tricky one. I'm gonna have to say it's the big pack there.

Researcher: Why is that then?
Liam: Well the big pack is 80p, now half price so 40p. But these packs cost 30 plus 30 plus 30 which is … 90 so yeah 40p.
Researcher: How much did you say they would be [small packs]?
Liam: 90p and you could buy 2 big packs for that!

I acknowledge the possibility that Liam may not have understood the term '3 packets for the price of 2' but his confidence in his final answer leads me to believe that he has simply not read or disregarded the information in the second star, leading him to get the correct answer, but for the wrong reasons. His mistakes here may be related to how the word problem is arranged on the page as all of the required information for the larger packet of pencils is in the star, but the information for the smaller packets is not.

Distracting Information

Two comments related to this category are shown below:

Liam: I like the little flashy.

Liam: Hah Patrick! Either you watch SpongeBob or you have a kid that watches SpongeBob.

Liam's first comment is related to the illustrations in the word problem in Figure 1 and his second to a word problem featuring a character called Patrick. Although Liam's observations and comments may not lead to incorrect answers, they show that his attention may not be focused on the mathematics required to answer the problem and therefore these distractions may cause him difficulties.

Giving an Answer that is Possible or Likely

Rachel was given a calculator to use to answer a word problem about the number of children going on a school trip and gave the answer of '8.333333 children'.

Rachel: 8 .333333 so 8.3 dot [recurring].
Researcher: Is that the number of children?

Rachel: Yeah.

Here, although Rachel had carried out an appropriate calculation, she has not given a correct answer and does not appear to realise, or consider it important that it is impossible to have a third of a child on a school trip.

Another example of a child experiencing difficulties within this category is below:

8. Kate bought a fizzy drink and a mars bar for £1.20. Sophie went to the same shop and bought a fizzy drink and 2 mars bars for £1.55. How much is a fizzy drink in that shop?

Liam: I can tell you this drink's gonna cost loads more than a Mars bar. It's 35 for the Mars bars so take away 35 which would be …that would be 85. It's £85. £85 for a fizzy drink.

Researcher: That sounds a lot doesn't it?

Liam: Mmmm [in agreement] I'm not going to this shop.

Researcher: Do you think that this sounds like a realistic shop then?

Liam: Yes cos I've seen things like my Pokemon cards and they cost £3.99 for one pack.

Researcher:	So you think that there could be a shop that sells a fizzy drink for £85?
Liam:	Yes.

Figure 2, a word problem given to Liam and the transcript of the conversation that followed.

Liam's difficulties are caused by problems transferring between pounds and pence, but his perception that £85 is a possible answer, even after attention was drawn to it, mean that he could not identify his error and went on to give an incorrect answer. This example is closely linked to the subcategory of 'Transferring an Answer into the Required Format' due to the difficulties he had with dealing with money. Conversely to this example, when answering an equivalent word problem to the one shown in Figure 2, Liam experienced similar difficulties with knowing when values were in pounds or pence, but was able to use his judgment of what is realistic to identify his error and go on to give a correct answer.

Transferring an Answer into the Required Format

The following excerpt shows Fiona having difficulties transferring her answer from a decimal number into money:

Fiona:	He has to pay 15 for four.
Researcher:	But it's half price isn't it so how much is that?
Fiona:	Uuuuh 7.5.
Researcher:	OK. What's that in money?
Fiona:	£7. 05.

Although Fiona was able to carry out the calculation of '15 ÷ 2 =' correctly, she had difficulties when trying to write that value in the standard format for money.

Other Findings

Other findings were also identified. These involved a child having difficulties because he was not using jottings. When prompted to write numbers down as he was calculating mentally, he was able to carry out a calculation more effectively.

Manipulatives were also shown to help a child to find a correct answer to a word problem after previously not being able to for equivalent word problems. This shows that manipulatives may be a useful tool for helping children to answer word problems marginally beyond their current grasp.

Children also made comments which suggested that they were able to identify the equivalence between word problems. This ability to recognise equivalence could imply that showing children how to correctly find an answer to a word problem may help those children to also solve equivalent word problems.

Recommendations

As a result of examining my findings alongside existing literature, I have compiled a list of strategies that teachers and researchers could trial to help children to overcome difficulties with mathematical word-problems:

- Encourage children to read the word-problems thoroughly;
- Teach children which kinds of information may be important;
- Ensure that children practise solving word-problems to allow them to be able to recognise the structure of word-problems and therefore know when to use each calculation;

- Consider giving children manipulatives to support the solving of word-problems currently beyond the scope of their ability;
- Encourage children to write down their workings so that they do not become unnecessarily confused;
- Encourage children to check if their answer satisfies the criteria of a question. For example if it is in the correct format;
- Teach children to calculate with monetary values;
- Encourage children to check if an answer is possible in the context of the question.

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