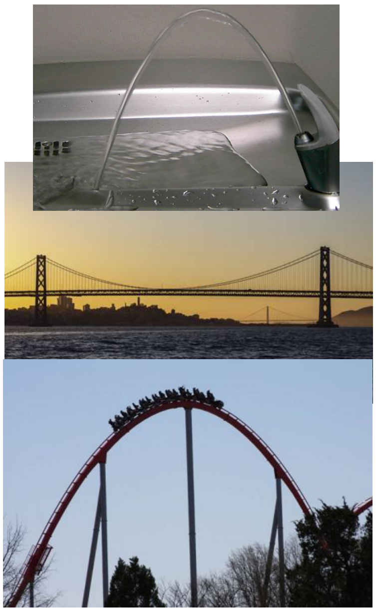
Word Problems in Vertex Form



Examples of quadratic functions are all around us and they all have their own distinct formula which we can put into vertex form.

height
h(t)

time

$$\boxed{y = a(x-p)^2 + q}$$

The values of x and y are often replaced with variables that represent the particular situation. For example, y may be replaced with h(t) and x with t.

$$\boxed{h(t) = -3(t-2)^2 + 5}$$

t represents a particular time

h(t) represents the height at that particular time

h(d) =

d =

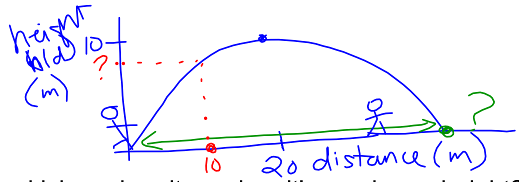
height

distance

Ex(1) The equation below shows the height of a soccer ball, h(d) meters as a function of the horizontal distance, d meters the ball travels until it hits the ground. The ball is kicked when it is on the ground.

$$h(d) = -0.025(d-20)^2 + 10$$

★ Always make a sketch to begin even if you are not asked to!!!



a) What is the maximum height of the ball?

10m

b) What is the horizontal distance of the ball from the kicker when it reaches it's maximum height?

20m

c) How far does the ball travel horizontally from when it is kicked until it hits the ground?

40m

$$h(d) = -0.025(d-20)^2 + 10$$

d) What is the height of the ball when it is 10 m horizontally from the kicker?

$h(d) = ?$

$d = 10$

$h(d) = -0.025(10-20)^2 + 10$

$h(d) = -0.025(100) + 10$

$= -2.5 + 10$

$= 7.5$

The hight is 7.5m

e) If ~~the~~ a player was 34 m away from the kicker and positioned under the ball, would he be able to head the ball? Explain.

$d = 34$

$h(d) = ?$

$h(d) = -0.025(34-20)^2 + 10$

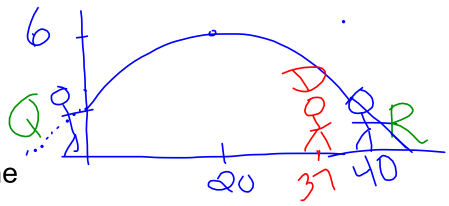
~~~ work :)

$h(d) = 5.1$

No, it is too high
~~~

Ex(2) A touch football quarterback passed the ball to a receiver 40m downfield. The path of the ball can be modelled by the following function.

$$h(d) = -0.01(d-20)^2 + 6$$



a) What was the maximum height of the ball?

6m

b) What was the horizontal distance of the ball from the quarterback when it reached its maximum height?

20m

c) What was the height of the ball when it was thrown?

$h(d) = ?$

$d = 0$

$$h(d) = -0.01(0-20)^2 + 6$$
$$= -0.01(400) + 6$$
$$= -4 + 6$$
$$= 2$$

$$h(d) = -0.01(d-20)^2 + 6$$

d) If the ball is not caught, what is the distance from the quarterback when the ball lands on the ground?

$h(d) = 0$
$d = ?$

$0 = -0.01(d-20)^2 + 6$ -6 -6

$\frac{-6}{-0.01} = \frac{-\cancel{0.01}(d-20)^2}{-\cancel{0.01}}$

$\sqrt{600} = \sqrt{(d-20)^2}$

$\pm 24.5 = d - 20$

$+24.5 = d - 20$ (+20, +20) OR $-24.5 = d - 20$ (+20, +20)

$\boxed{44.5 = d}$ $-4.5 = d$

e) If a defensive back is 3m in front of the receiver, would he be able to jump up and knock down the pass?

$d = 37$
$h(d) = ?$

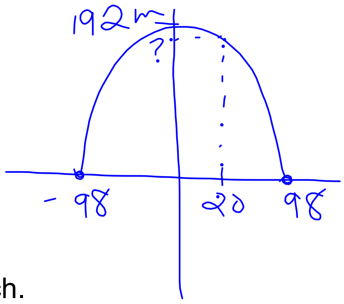
work . . .

$h(d) = 3.11\text{m}$

Ex(3): The Gateway arch in St. Louis has a shape that approximates a parabola. The curve can be modelled by the following function: $$h(d) = -0.02d^2 + 192$$

where *h(d)* is the height in meters and *d* is the horizontal distance from the centre of the arch.

a) Sketch and label a graph of the arch



b) Determine the maximum height of the arch.

192m

c) determine the height of the arch 20 m from the centre. $h(d) = -0.02d^2 + 192$

$d = 20$
$h(d) = ?$

$h(20) = -0.02(20)^2 + 192$
$= -0.02(400) + 192$
$= -8 + 192$
$\boxed{h(20) = 184\text{m}}$

The arch is 184m high 20m from centre

d) Determine the overall width of the arch.

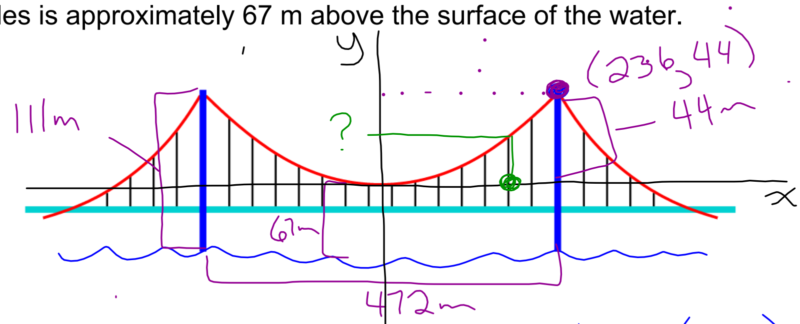
$d = ?$
$h(d) = 0$

$0 - 192 = -0.02d^2 + 192 - 192$
$\frac{-192}{-0.02} = \frac{-0.02d^2}{-0.02}$
$\sqrt{9600} = \sqrt{d^2}$
$\pm 97.98 = d$

It is approx 98m on each side, so a total width of 196m.

Ex(4) The deck of the Lion's Gate Bridge in Vancouver is suspended from two main cables attached to the tops of two supporting towers. Between the towers, the main cables take the shape of a parabola as they support the weight of the deck. The towers are 111 m tall relative to the water's surface and are 472 m apart. The lowest point of the cables is approximately 67 m above the surface of the water.

a) Label the diagram.



b) Model the shape of the cables with a quadratic function in vertex form.

vertex (0, 0)

point (236, 44)

$y = a(x-p)^2 + q$

$44 = a(236-0)^2 + 0$

$a = 0.00079$

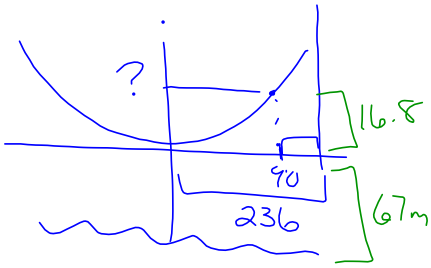
$$\boxed{\begin{aligned} f(x) &= 0.00079x^2 \\ h(d) &= 0.00079d^2 \end{aligned}}$$

c) Determine the height above the surface of the water of a point on the cables that is 90 m horizontally from one of the towers.

$d = \cancel{90}\ 146$

$h(d) = ?$

$h(d) = 0.00079d^2$



$h(146) = 0.00079(146)^2$

$h(d) = 16.8$

$+67$

83.8

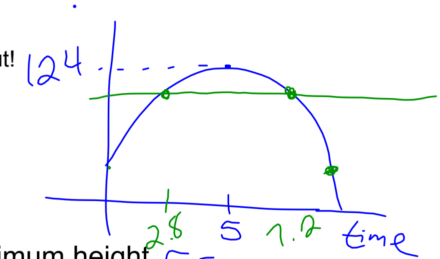
You Try: The path of a firework is described by the function: $h(t) = -4.9(t-5)^2 + 124$

where h(t) is the height of the firework, in meters, and t is the time in seconds, since the launch.

a) Sketch the pathway of the firework.
Is it launched from the ground or a platform?? Determine the height at time 0 to find out!

$h(t) = -4.9(0-5)^2 + 124$

1.5m



b) Determine the maximum height of the firework. 124m

c) How many seconds does it take for the firework to reach the maximum height. 5s.

d) Determine the height 3.5 seconds after the firework is launched. $t = 3.5 \rightarrow h(t) = 113$

e) How many seconds does it take for the firework to reach a height of 100 m

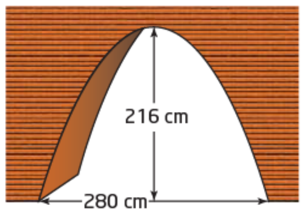
f) For how long is the firework in the air? $0 = -4.9(t-5)^2 + 124$

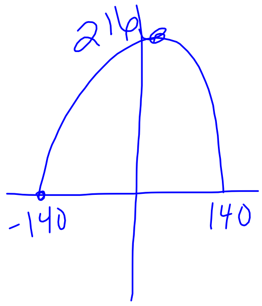
Your Turn

Suppose a parabolic archway has a width of 280 cm and a height of 216 cm at its highest point above the floor.

a) Write a quadratic function in vertex form that models the shape of this archway.

b) Determine the height of the archway at a point that is 50 cm from its outer edge.





Vertex $(0, 216)$ p, q point $(140, 0)$ x, y

$$0 = a(140-0)^2 + 216$$
-216 -216

$$\frac{-216}{19600} = \frac{a(19600)}{19600}$$

$$a = -\frac{216}{19600} = -\frac{54}{4900} = -\frac{27}{2450}$$

a) $\boxed{h(d) = -\frac{27}{2450}d^2 + 216}$

b.) $h(d) = -\frac{27}{2450}(90)^2 + 216$

$\boxed{h(d) = 126.7\text{cm}}$

* 50 cm from edge = 90 from (0,0)

Worksheet to practice

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